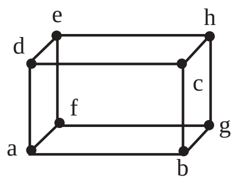
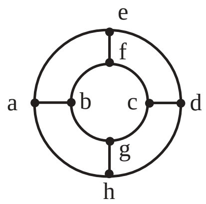
(e) Show that the set of real numbers of the form $a + b\sqrt{2}$ where a and b are integers with ordinary addition and multiplication is not a field.

(f) Find the solution of the equation a.b.x.a.x = c.b.x in a group G, where a, b, c are given element of G.

5. Attempt any **two** parts of the following : **(2×10=20)**

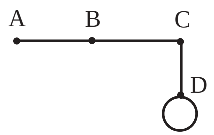
(a) Define isomorphic graphs. Find out whether the following two graphs are isomorphic or not :

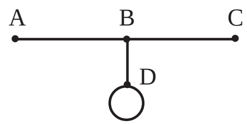




Explain your answer.

(b) Define with one example (i) Eulerian path and (ii) Eulerian circuit. Also find out which of the following graph is an Eulerian path





Explain your answer

(c) Explain what do you understand by finite state machine. Design a finite state automation that accepts those strings over {0, 1} such that the number of zeros is divisible by 3.

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. IV) EVEN THEORY EXAMINATION 2012-13

DISCRETE MATHEMATICS

Time : 3 Hours *Total Marks : 100*

Note :– Attempt **all** the questions.

1. Attempt any **four** parts of the following : **(4×5=20)**

(a) Let A and B be two sets then prove that :

$$A \Delta B = (A - B) \cup (B - A)$$

(b) In a survey of 180 people, it was observed that 30 people read Hindustan Times, 25 read Times of India, 28 read The Tribune, 15 read both Hindustan Times and The Tribune, 18 read both Times of India and The Tribune, 20 read both Hindustan Times and Times of India and 5 read all three papers. Find :

(i) The number of people who read at least one of the three newspapers.

(ii) The number of people who read no newspaper at all.

(c) Let $f : A \to B$ and $g : B \to C$ be two functions, then show that :

(i) $g \circ f : A \to C$ is onto if f and g are onto

(ii) $g \circ f : A \to C$ is one-to-one if f and g are one-to-one.

(d) If $f : A \to B$ and $g : B \to C$ are bijective mappings, then show that :

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

(e) Let S be the set of all points in a plane. Let R be a relation such that for any two points, a and b; $(a, b) \in R$ if b is within two centimeters from a, show that R is an equivalence relation.

(f) Define a relation R on a set $X = \{1, 2, 3, 4, 5\}$:

(i) Which is only reflexive.

(ii) Which is reflexive and symmetric.

(iii) Which is symmetric but not reflexive.

2. Attempt any **four** parts of the following : **(4×5=20)**

(a) Show, by mathematical induction, that the sum of first n odd numbers is n^2.

(b) Using truth table, find out whether following logical expressions are equivalent or not ?

$$p \wedge (q \vee r) \text{ and } (p \wedge q) \vee (p \wedge r)$$

(c) Using algebra of proposition, show that :

$$p \Leftrightarrow q \equiv (p \vee q) \Rightarrow (p \wedge q)$$

(d) Obtain the principal disjunctive normal form of :

$$(p \wedge \sim q \wedge \sim r) \vee (q \wedge r)$$

(e) Find out whether the following proposition is tautolog or not ?

$$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$$

(f) Check the validity of the following argument :

"If Roli has completed MCA, then she is assured a good job.

If Roli is assured a good job, she is happy.

Roli is not happy.

Therefore, Roli has not completed MCA."

3. Attempt any **two** parts of the following : **(2×10=20)**

(a) (i) Prove that $P(n, r) = P(n-1, r) + r\,P(n-1, r-1)$

(ii) Prove that ${}^{n}c_{r} = {}^{n+1}c_{r} - {}^{n}c_{r-1}$

(b) Find the general solution of following recurrence relation :

$$a_r - a_{r-1} - 12a_{r-2} = (-3)^r + 6.4^r$$

give $a_0 = 2$ and $a_1 = 8$.

(c) Find the general solution of the following recurrence relation, using generating function :

$$a_r - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0, \; n \geq 3$$

with $a_0 = 0$, $a_1 = 1$ and $a_2 = 10$.

4. Attempt any **four** parts of the following : **(4×5=20)**

(a) Show that the set Q^+ of positive rational numbers does not form a group for the composition $*$ defined by $a * b = a/b \; \forall \; a, b \in Q^+$.

(b) Show that the set $Q - \{1\}$ of rational numbers other that 1 is an abelian group under the composition $*$ defined as

$$x * y = x + y - xy$$

(c) Show that the set X of four permutations I, (1 2) (3 4), (1 3) (2 4) and (1 4) (2 3) on four symbols 1, 2, 3, 4 is an abelian group with respect to the permutation multiplication.

(d) Let R be a system satisfying all the postulates required for a ring with the possible exception of $a + b = b + a$. If there exists one element $c \in R$ such that $ac = bc \Rightarrow a = b \; \forall \; a, b \in R$, then prove that R is a ring.