(d) Determine whether a semi group with more than one idempotent elements can be a group.

(e) Prove that the intersection of any two subgroups of a group G is again a subgroup of G.

(f) Define a ring. Show that the system [E, +, ×) of even integers is a ring under ordinary addition and multiplication.

5. Attempt any **TWO** parts of the following :— **(10×2=20)**

(a) Show that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

(b) Write short notes on the following :—

(i) Planer graph

(ii) Euler graph

(iii) Graph Coloring

(c) (i) Given the in order and post order traversal of a tree T :

In order : BEHFACDGI

Post order : HFEABIGDC

Determine the tree T and its pre order.

(ii) Prove that a binary tree with n vertices has exactly (n+1) null branches.

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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934 **Roll No.**

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2013-14

DISCRETE MATHEMATICS

Time : 3 Hours *Total Marks : 100*

Note :— *(1) Attempt **ALL** questions.*

(2) Each question carries equal marks.

1. Attempt any **FOUR** parts of the following :— **(5×4=20)**

(a) If A and B are two sets, prove that

$A \cup B = (A - B) \cup B$

(b) Prove that union of two countably infinite set is countably infinite.

(c) If R is an equivalence relation on a Set A, then show that R^{-1} is also an equivalence relation on A.

(d) Let X = {1, 2, 3, 4} and R = {(x, y) : x > y}.

(i) Give the ordered pair of R.

(ii) Draw the graph of R.

(iii) Give the relation matrix of R.

(e) Let A = {1, 2, 3, 4}. Give an example of R in A which is :

(i) neither symmetric nor reflexive

(ii) symmetric, reflexive but not transitive

(iii) transitive and reflexive but not symmetric.

(f) Let f, g, h $\in$ R be defined as

$f(x) = x - 5$, $g(x) = x + 5$, $h(x) = 7x \ \forall \ x \in R$.

Find $g \circ f$, $h \circ f$, $f \circ h \circ g$.

2. Attempt any **TWO** parts of the following :— **(10×2=20)**

(a) (i) Find whether the proposition $P \vee \sim (p \wedge q)$ is tautology or not.

(ii) Find whether the proposition $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction or not.

(b) (i) The contrapositive of a statement S is given as "If $x < 2$ then $x + 4 < 6$." Write the statement S and its converse.

(ii) Find a formula A that uses the variable p, q and r such that A is a contradiction.

(c) (i) Translate the following statements in quantified expressions of predicate logic :

(α) all students need financial aid.

(β) some students need financial aid.

(ii) Determine the validity of the following argument using truth table :

S_1 : If 7 is less than 4, then 7 is not a prime number.

S_2 : 7 is not less than 4.

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S : 7 is a prime number.

3. Attempt any **TWO** parts of the following :— **(10×2=20)**

(a) (i) Find the simple expression for the generating function of following discrete numeric function :

$$1, \frac{2}{3}, \frac{3}{9}, \frac{4}{27}, \ldots\ldots\ldots, \frac{r+1}{3^r}, \ldots\ldots$$

(ii) Solve the recurrence relation

$a_r - 6a_{r-1} + 8a_{r-2} = r.4^r$, given $a_0 = 8$, $a_1 = 22$.

(b) (i) A committee of 5 is to be formed out of 6 boys and 4 girls. In how many ways this can be done when at least 2 girls are included ?

(ii) How many selections any number at a time may be made from three white balls, four green balls, one red ball and one black ball if at least one must be chosen.

(c) Using generating function, solve the following recurrence relation :

$$a_r - 9a_{r-1} + 26\, a_{r-2} - 24\, a_{r-3} = 0,\ n \geq 3$$

with $a_0 = 0$, $a_1 = 1$ and $a_2 = 10$.

4. Attempt any **FOUR** parts of the following : **(5×4=20)**

(a) Define a group. Verify whether the set of all integers Z forms a group with respect to difference.

(b) Define a cyclic group. Prove that cyclic group is abelian.

(c) Show that the set N of natural numbers is a semigroup under the operation $x * y = \max(x \cdot y)$. Is it a monoid ?