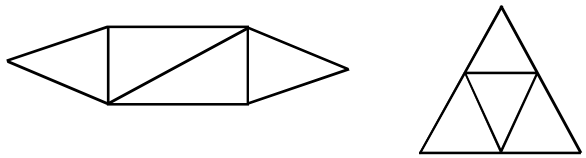
(ii) For the following pair of graphs, determine whether or not the graphs are isomorphic. Explain your answer :



(b) (i) Define spanning tree in a graph. Find all the spanning trees of the graph given in Q.No. 5. (a) ii.

(ii) Discuss Chinese Postman Problem.

(c) What do you understand by Finite Automation ? For the finite state machine whose transition function δ is given in the following table and

$Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = (0,1)$ $F = \{q_0\}$, given the entire sequence of states for the input string 110101.

States	Inputs 0	1
q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199414 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. IV) THEORY EXAMINATION 2013-14

DISCRETE MATHEMATICS

Time : 3 Hours *Total Marks : 100*

Note :– Attempt **all** questions. All questions carry equal marks.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Define reflexive, symmetric and transitive relations. Let $A = \{1,2,3,4\}$ give an example of R in A which is

(i) Reflexive and transitive but not symmetric.

(ii) Neither reflexive nor transitive but symmetric.

(b) Let $f : R\rightarrow R$ be defined by $f(x) = x^2$ and $g : R\rightarrow R$ be defined by $g(x) = x+3$, then find out $(fog): R\rightarrow R$ and $(gof): R\rightarrow R$. Also show whether composite functions are equal.

(c) Let S be the set of all points in a plane. Let R be a relation such that for any two points, a and b; $(a, b) \in R$ if b is within two centimeters from a, show that R is an equivalence relation.

(d) What is meant by recursively defined function ? Give the recursive definition of factorial function.

(e) Let A, B, C be sub sets of U. Given the $A \cap B = A \cap C$, is it necessary that $B = C$? Justify your answer.

(f) Let A = {ϕ, b}, construct the following sets :

(i) A – {ϕ}

(ii) {ϕ} – A

(iii) A ∪ P (A)

(iv) A ∩ P (A)

(v) P (A) – A

2. Attempt any **four** parts of the following : **(5×4=20)**

(a) Prove that the statement formulae

$(p \to q)$ and $(\sim p \vee q)$ are equivalent.

(b) Find out whether the following statement

$((p \vee q) \wedge \sim (\sim p \wedge (\sim q \vee \sim r))) \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$ is a tautology or not.

(c) Test whether the following argument is valid or not :

"If 4 is not even, then 3 is not prime.

But 4 is even

....

Therefore, 3 is prime."

(d) Express $(\sim p \to r) \wedge (q \leftrightarrow q)$ in its principle conjunctive normal form.

(e) Using logical equivalent formulas, show that $\sim (p \vee (\sim p \wedge q) \cong \sim p \wedge \sim q$.

(f) Using logical equivalent formula prove that the following implication is a tautoiogy :

$(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) How many trianlges are determined by the vertices of a regular polygon of n sides ? How many of them are if no sides of the polygon is to be a side of any triangle ?

(ii) In how many way can the symbols a,b,c,d,e,e,e,e,e be arranged so that no e is adjacent to another e.

(b) Using generating function, solve

$a_{n+2} - 5a_{n+1} + 6a_n = 2, n \geq 0$ and $a_0 = 3, a_1 = 7$

(c) Solve the recurrence relation

$a_r - 2a_{r-1} + a_{r-2} = 2^r$

With the conditions $a_0 = 2$ and $a_1 = 1$

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) Define the abelian group. Consider the operator * defined on Z, the set of integers. as

$x * y = x + y + 1 \ \forall \ x, y$ € Z, show that (Z, *) is an abelian group.

(b) Define a field. Show that the system of even integers is a ring under ordinary addition and multiplication.

(c) Let u(8) = {1,3,5,7} be a group with respect to multiplication modulo 8. Also show that every element of u(8) is its own inverse.

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) Define the degree of a vertex in a graph. Prove that the sum of the degrees of all the vertices of a grap is twice of the number of edges in graph.