LAGRANGE'S INTERPOLATION FORMULA

Let $f(x_0), f(x_1), \ldots\ldots, f(x_n)$ be $(n + 1)$ entries of a function $y = f(x)$, where $f(x)$ is assumed to be a polynomial corresponding to the arguments $x_0, x_1, x_2, \ldots\ldots, x_n$.

The polynomial $f(x)$ may be written as

$$f(x) = A_0 (x - x_1)(x - x_2) \ldots\ldots (x - x_n)$$
$$+ A_1(x - x_0)(x - x_2) \ldots\ldots (x - x_n)$$
$$+ \ldots\ldots + A_n (x - x_0)(x - x_1) \ldots\ldots (x - x_{n-1})$$

where $A_0, A_1, \ldots\ldots, A_n$ are constants to be determined.

Putting $x = x_0, x_1, \ldots\ldots, x_n$ in (50), we get

$$f(x_0) = A_0 (x_0 - x_1)(x_0 - x_2) \ldots\ldots (x_0 - x_n)$$

$$\therefore \quad A_0 = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2) \ldots\ldots (x_0 - x_n)}$$

$$f(x_1) = A_1 (x_1 - x_0)(x_1 - x_2) \ldots\ldots (x_1 - x_n)$$

$$\therefore \quad A_1 = \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2) \ldots\ldots (x_1 - x_n)}$$

$$\vdots \qquad \vdots \qquad \vdots$$

Similarly, $A_n = \dfrac{f(x_n)}{(x_n - x_0)(x_n - x_1) \ldots\ldots (x_n - x_{n-1})}$

Substituting the values of $A_0, A_1, \ldots\ldots, A_n$ in equation (50), we get

$$f(x) = \frac{(x - x_1)(x - x_2) \ldots\ldots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) \ldots\ldots (x_0 - x_n)} f(x_0)$$
$$+ \frac{(x - x_0)(x - x_2) \ldots\ldots (x - x_n)}{(x_1 - x_0)(x_1 - x_2) \ldots\ldots (x_1 - x_n)} f(x_1)$$
$$+ \ldots\ldots + \frac{(x - x_0)(x - x_1) \ldots\ldots (x - x_{n-1})}{(x_n - x_0)(x_n - x_1) \ldots\ldots (x_n - x_{n-1})} f(x_n)$$

This is called **Lagrange's Interpolation Formula.**

Example 1. *Using Lagrange's interpolation formula, find y(10) from t following table:*

x	5	6	9	11
y	12	13	14	16

Sol. Here $x_0 = 5$, $x_1 = 6$, $x_2 = 9$, $x_3 = 11$

$f(x_0) = 12$, $f(x_1) = 13$, $f(x_2) = 14$, $f(x_3) = 16$

Lagrange's formula is

$$f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} f(x_0)$$

$$+ \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} f(x_1)$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} f(x_2)$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} f(x_3)$$

$$f(x) = \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)}(12)$$

$$+ \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)}(13)$$

$$+ \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)}(14)$$

$$+ \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)}(16)$$

$$= -\frac{1}{2}(x-6)(x-9)(x-11) + \frac{13}{15}(x-5)(x-9)(x-11)$$

$$- \frac{7}{12}(x-5)(x-6)(x-11)$$

$$+ \frac{4}{15}(x-5)(x-6)(x-9)$$

Putting $x = 10$, we get

$$f(10) = -\frac{1}{2}(10-6)(10-9)(10-11) + \frac{13}{15}(10-5)(10-9)(10-11)$$

$$- \frac{7}{12}(10-5)(10-6)(10-11) + \frac{4}{15}(10-5)(10-6)(10-9)$$

$$= 14.66666667$$

Hence,

$y(10) = 14.66666667.$