5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Define following with one example :

(i) Regular graph.

(ii) Complete graph.

(iii) Bi-partite graph.

(iv) Hamiltonian path.

(v) Chromatic number.

(b) (i) If G is a non-trivial tree, then prove that G contains at least two vertices of degree one.

(ii) Define binary tree and discuss two important applications of it.

(c) What do you understand by Automation theory ? For the finite state machine whose transition function δ is given in the table and

$Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = (0, 1)$, $F = \{q_0\}$,

give the entire sequence of states for the input string 110101.

States	Inputs	
	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934 **Roll No.**

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2012-13

DISCRETE MATHEMATICS

Time : 3 Hours *Total Marks : 100*

Note : *Attempt **all** questions.*

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Draw a Venn diagram of sets A, B, C where :

(i) A and B have elements in common, B and C have elements in common, but A and C are disjoint.

(ii) $A \leq B$, set A and C are disjoint, but B and C have elements in common.

(b) Let $f : R \to R$ and $g : R \to R$, where R is the set of real numbers. Find f o g and g o f, where $f(x) = x^2 - 2$ and $g(x) = x + 4$. State where these functions are injective, surjective or bijective.

(c) If R and S are equivalance relations on the set A, show that the following are equivalence relations :

(i) $R \cap S$

(ii) $R \cup S$

(d) Let $A = \{1, 2, 3\}$. Define $f : A \to A$ such that $f = \{(1, 2), (2, 1), (3, 3)\}$. Find :

(i) f^{-1} (ii) f^2 (iii) f^3.

(e) Let S be the set of all points in a plane. Let R be a relation such that for any two points, a and b;

$(a, b) \in R$ if be is within two centimetre from a, show that R is an equivalence relation.

(f) What is meant by recursively defined function ? Give the recursive definition of factorial function.

2. Attempt any **four** parts of the following : **(5×4=20)**

(a) Find whether the implication is tautology or not :

$$((P \vee \sim Q) \wedge (\sim P \vee \sim Q)) \vee Q.$$

(b) Draw the truth table for the statement :

$$(\sim(P \vee Q)) \vee ((\sim P) \wedge Q).$$

(c) Determine whether the following argument is valid or not :

"If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number."

(d) Using truth table prove that :

$$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P).$$

(e) Using logical equivalent formulas, show that

$$\sim (P \vee (\sim P \wedge Q)) \equiv \sim P \wedge \sim Q.$$

(f) Using logical equivalent formulas prove that the following implication is a tautology :

$$(P \to (Q \to R)) \to ((P \to Q) \to (P \to R)).$$

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) How many triangles are determined by the vertices of a regular polygon of n sides ? How many of them are if no side of the polygon is to be a side of any triangle ?

(ii) In how many ways can the symbols a, b, c, d, e, e, e, e, e be arranged so that no e is adjacent to another e.

(b) Solve the recurrence relation given below :

$$a_r - 7a_{r-1} + 10a_{r-2} = 6 + 8r,\ n \geq 2$$

given $a_0 = 1$ and $a_1 = 2$.

(c) Using generating function, solve the following recurrence relation :

$$a_r - 9a_{r-1} + 26a_{r-2} - 24a_{r-3} = 0,\ n \geq 3$$

with $a_0 = 0$, $a_1 = 1$ and $a_2 = 10$.

4. Attempt any **four** parts of the following : **(5×4=20)**

(a) Define a group. Verify whether the set of all 2×2 matrices of real numbers form a group with respect to matrix multiplication.

(b) Show that the system $(E, +, \cdot)$ of even integers is a ring under ordinary addition and multiplication.

(c) Let G be the set of all nonzero real numbers and let

$$a * b = \frac{ab}{2}.$$

Show that $(G, *)$ is an abelian group.

(d) Let $u(8) = \{1, 3, 5, 7\}$ be a group with respect to multiplication modulo 8. Prove that every element of u(8) is its own inverse.

(e) Prove that the union of two subgroups of a group G is a subgroup if and only if one is contained in the other.

(f) If A_n is the set of all even permutations of degree n, then prove that A_n is a finite group of order $\frac{n!}{2}$ with respect to product of permutations.