GALGOTIAS COLLEGE OF ENGINEERING AND TECHNOLOGY

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Applied Sciences & Humanities Department

Session: 2017-18 **Semester: III** **Section: EI B &ICE**
Subject Code: ROE 038 **Subject Name: Discrete Mathematics**

Assignment 1

Note: 1. Submit your assignment in a copy/register, assignment in the form of individual pages/A4 is not acceptable.

1. Prove that the statement formulae
   (a) $(p \to q)$ and $(\sim p \vee q)$ are equivalent.
   (b) Find out whether the following statement
   $((p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))) \wedge (\sim p \wedge \sim q) \wedge (\sim p \wedge \sim r)$ is a tautology or not.
   (c) Test whether the following argument is valid or not:
   (i) "If 4 is not even, then 3 is not prime. But 4 is even
   .... Therefore, 3 is prime".
   (ii) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.
2. Justify the following.
   (a) Express $(\sim p \to r) \wedge (q \leftrightarrow q)$ in its principle conjunctive normal form.
   (b) Using logical equivalent formulas, show that: $\sim (p \vee (\sim p \wedge q) \cong \sim p \wedge \sim q$.
   (c) Using logical equivalent formula prove that the following implication is a tautology:
   $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.
   (d) What is meant by recursively defined function? Give the recursive definition of factorial function.
3. (a) Let $f : R \to R$ and $g : R \to R$, where R is the set of real numbers. Find $f\ o\ g$ and $g\ o\ f$, where $f(x) = x^2 - 2$ and $g(x) = x + 4$. State where these functions are injective, surjective or bijective.
   (b) If R and S are equivalence relations on the set A, show that the following are equivalence relations: (i) $R \cap S$ (ii) $R \cup S$.
   (c) Let $A = \{1, 2, 3\}$. Define $f : A \to A$ such that $f = \{(1, 2), (2, 1), (3, 3)\}$. Find
   (i) f^{-1} (ii) f^2 (iii) f^3.
4. (a) If N is the set of natural numbers and R is a relation in $N \times N$ defined as $(a, b)\ R\ (c, d)$ if and only if $ad = bc$, then prove that R is an equivalence relation.
   (b) The function f maps a calendar month onto the number of days in that month. What is the range of function f when the domain is: (i) {months in 1979} (ii) {months in 1980}
   (c) If the relation R in the set of all natural numbers is defined by "aRb if and only if $a^2 - 4ab + 3b^2 = 0$". Prove that R is reflexive and it is neither symmetric nor transitive.
   (d) How many total number of distinct relations from a set A and a set B can be found if A contains m and B contains n elements?
5. Attempt any two parts of the following:
   (a) Construct the truth table: (i) $\sim(P \wedge Q) \leftrightarrow P \vee Q$ (ii) $(P \to Q) \leftrightarrow (Q \to P)$
   (b) Show that: $\big((P \vee \sim Q) \wedge (\sim P \vee \sim Q)\big) \vee Q$ is a tautology using definition.
   (c) Test the validity of the following argument:
   "If Ashok wins then Ram will be happy. If Kamal wins, Raju will be happy. Either Ashok will win or Kamal will win. However, if Ashok wins, Raju will not be happy and if Kamal wins, Ram will not be happy. So Ram will be happy iff Raju is not happy
6. (a) Obtain the principal disjunctive normal form of :$(p \wedge \sim q \wedge \sim r) \vee (q \wedge r)$.
   (b) Using truth table, find out whether following logical expressions are equivalent or not?
   $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$.
   (c) If $f : A \to B$ and $g : B \to C$ are bijective mappings, then show that: $(g\ o f)^{-1} = f^{-1} o g^{-1}$
   (d) Prove that union of two countably infinite set is countably infinite.
9. (a) If R is an equivalence relation on a Set A, then show that R^{-1} is also an equivalence relation on A.
   (b) Let $X = \{1, 2, 3, 4\}$and $R = \{(x, y): x > y\}$.
   (i) Give the ordered pair of R.
   (ii) Draw the graph of R.
   (iii) Give the relation matrix of R.
10. Let $A = \{1, 2, 3, 4\}$. Give an example of R in A which is :
    a. Neither symmetric nor reflexive
    b. Symmetric, reflexive but not transitive
    c. Transitive and reflexive but not symmetric.